The influence of piedmont deposition on the time scale of mountain-belt denudation

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[1] The linear correlation between modern sediment yields and mean drainage-basin elevation suggests that mountain-belt topography is denuded exponentially with a time scale of approximately 50 Myr following the cessation of active uplift. Some Paleozoic orogens, however, still exist as high-elevation terrain. Here I explore this paradox within the broader question of what variables control the denudational time scales of mountain belts. Using a two-dimensional model that couples the stream-power law for bedrock channel erosion with the diffusion equation for alluvial piedmonts, I determined the time scale of mountain-belt denudation using numerical and analytic techniques. The piedmont plays an important role in mountain-belt denudation because it sets the base level for bedrock erosion, substantially reducing bedrock relief in mountain belts with broad or steep piedmonts. The persistence of the Appalachian and Ural Mountains may be understood within the model framework as largely the result of resistant bedrock and a broad piedmont, respectively. *INDEX TERMS:* 1824 Hydrology: Geomorphology (1625); 3210 Mathematical Geophysics: Modeling; 3230 Mathematical Geophysics: Numerical solutions; 8102 Tectonophysics: Continental contractional orogenic belts. **Citation:** Pelletier, J. D. (2004), The influence of piedmont deposition on the time scale of mountain-belt denudation, *Geophys. Res. Lett.*, *31*, L15502, doi:10.1029/2004GL020052.

1. Introduction

[2] A great deal of interest has been focused on landscape evolution in mountain belts. Several state-of-the-art, three-dimensional numerical models have been developed that incorporate landsliding and bedrock channel erosion, for example [e.g., *Howard*, 1994; *Braun and Sambridge*, 1997; *Densmore et al.*, 1998; *Willett*, 1999]. While these models capture erosion in much of its true complexity, piedmont deposition is generally neglected and eroded sediment is assumed to be transported immediately out of the drainage basin. Can piedmont deposition strongly influence the morphology and denudation of mountain belts?

[3] Here I address this question within the context of a classic problem: the persistence of ancient mountain-belt topography. Global sediment yields are linearly correlated with mean drainage-basin elevation. In areas of old continental crust, *Pinet and Souriau* [1988] obtained

$$E = 0.61 \times 10^{-7} H \tag{1}$$

where E is the erosion rate in m/yr and H is the mean drainage-basin elevation in m. Many complimentary studies have documented linear correlations between erosion rates and mean drainage-basin elevation, relief ratio, and drainage-basin slope both within and between ranges [e.g., *Ruxton and McDougall*, 1967; *Ahnert*, 1970; *Summerfield and Hulton*, 1994]. Modern erosion rates determined from sediment yields do not necessarily equal erosion rates over geologic time scales [*Kirchner et al.*, 2001; *Zhang et al.*, 2001], but cosmogenic nuclide measurements prove that modern and ancient erosion rates are comparable in the southern Appalachians [*Matmon et al.*, 2003].

[4] Equation (1) can also be expressed as a differential equation for mountain-belt topography following the cessation of active uplift [*Ahnert*, 1970]:

$$\frac{\partial H}{\partial t} = -\frac{1}{\tau_d} H \tag{2}$$

with $\tau_d = 16$ Myr using (1). Equation (2) has the solution $H = H_0 e^{-t/\tau_d}$, where H_0 is the mean drainage-basin elevation immediately following the cessation of active uplift. Airy isostasy will increase τ_d by a factor of $\rho_c/(\rho_m - \rho_c)$, where ρ_m and ρ_c are the densities of the mantle and crust, but the waning buoyancy of ancient mountain belts results in a somewhat smaller increase [*Fischer*, 2002]. With isostasy included, $\tau_d \approx 45–70$ Myr using (1) and (2) and assuming an Airy isostatic ratio of between 3 and 4.5. This range of values for τ_d is about 5 times smaller than the age of Paleozoic orogens, several of which stand to well over 1 km in peak elevation.

[5] *Baldwin et al.* [2003] performed numerical experiments using the stream-power law to investigate this paradox and relate the denudational time scale to specific parameters of that law. The stream-power law in its general, two-dimensional form is given by

$$\frac{\partial h}{\partial t} = -Kx^{2m}\left|\frac{\partial h}{\partial x}\right|^n + K'\tau_c^a \tag{3}$$

where h is the local elevation, x is the distance along the longitudinal profile, K is the bedrock erodibility, m and n are empirical constants, and K', τ_c, and a are additional parameters related to a threshold shear stress for erosion [*Whipple and Tucker*, 1999]. *Baldwin et al.* [2003] argued that nonlinearity in the stream-power law (i.e., $n > 1$), a transition to transport-limited conditions in older mountain belts (i.e., piedmont deposition), and a finite threshold shear stress were all likely reasons for the persistent topography of some ancient orogens.

2. Model

[6] In principle, m, n, and τ_c may take on any values, but slope-area relationships for bedrock drainage basins indicate

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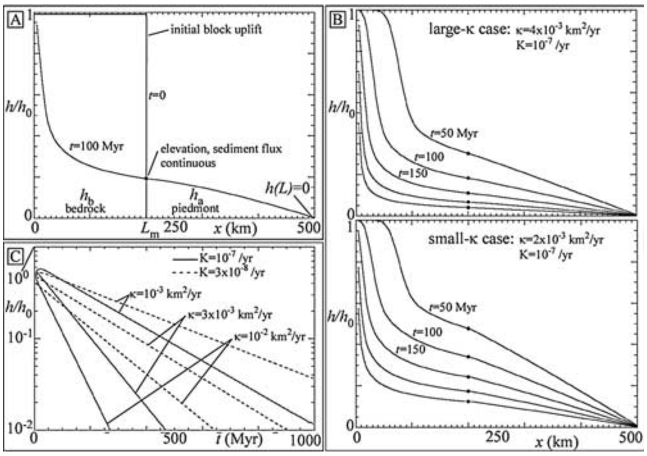

Figure 1. (A) Model geometry and key variables. (B) Plots of elevation vs. downstream distance for two values of κ. Each plot shows a temporal sequence from t = 50 Myr–250 Myr in 50 Myr intervals. (C) Plots of mean drainage-basin elevation vs. time for L_m = 100 km, L = 500 km, and a range of values of K and κ.

that $m/n \approx 0.5$ [*Whipple and Tucker*, 1999; *Snyder et al.*, 2000] and the linear correlations between erosion rates and mean drainage-basin elevation, relief ratio, and drainage-basin slope (with zero intercept; e.g., equation (1)) strongly suggest that, on average, $n \approx 1$ and $\tau_c \approx 0$. Using these values, the stream-power law becomes

$$\frac{\partial h}{\partial t} = \frac{K}{A_r} x \frac{\partial h}{\partial x} \tag{4}$$

with Airy isostatic uplift included ($A_r = \rho_c/(\rho_m - \rho_c)$), and assuming $x = A^{\frac{1}{2}}$. *Baldwin et al.* [2003] showed that piedmont deposition could significantly influence the time scale of mountain-belt denudation. To further quantify this control for specific piedmont geometries, I constructed a model that couples equation (4) to the diffusion model for alluvial transport [*Paola et al.*, 1992]:

$$\frac{\partial h}{\partial t} = \kappa \frac{\partial^2 h}{\partial x^2} \tag{5}$$

where κ is the diffusivity of sediment on the piedmont. A similar model was used by *Humphrey and Heller* [1995] for cyclic sedimentation.

[7] The model geometry and boundary conditions are illustrated in Figure 1a. h_a and h_b refer to the alluvial and bedrock portions of the profile. Tectonic uplift is assumed to occur as a rigid block between $x = 0$ and $x = L_m$ prior to $t = 0$. h_0 is the maximum elevation immediately following the cessation of active uplift. This boundary condition is applied a small distance L_h from the divide (i.e., the hillslope length) to give $h_b(L_h, 0) = h_0$. Sea level is assumed to be constant, giving $h_a(L, t) = 0$. The two remaining boundary conditions are continuity of elevation and sediment flux at the mountain front, $x = L_m$:

$$h_b(L_m) = h_a(L_m), \text{ and} \tag{6}$$

$$\int_{L_h}^{L_m} A_r \frac{\partial h_b}{\partial t} dx = \kappa \left[\frac{\partial h_a}{\partial x} \right|_{x=L_m}. \tag{7}$$

[8] The model makes a number of simplifying assumptions. First, the lateral position of the bedrock-alluvial boundary is assumed to be constant. As tectonic activity and sediment supply decreases, piedmonts adjacent to older mountain belts can become stripped of alluvial cover and develop pediment surfaces. This could be treated as a moving-boundary problem [e.g., *Swenson et al.*, 2000] within a more comprehensive model. Second, foreland-basin subsidence is not explicitly included, but subsidence can be considered implicitly in the model by varying the Airy isostatic ratio to include storage within the foredeep. Third, as a two-dimensional model, drainage reorganization and other three-dimensional effects are not considered. In the Appalachians, for example, there is thermochronologic evidence for a Miocene drainage reversal [*Naeser et al.*, 2001] that may have led to episodic denudation. River capture is one reason why any model that predicts steady, monotonic denudation, is not correct in detail. The advantage of a simple, two-dimensional model, however, is that an analytic result can be obtained that relates the time scale of mountain-belt denudation to fundamental controlling parameters through a basic set of equations.

3. Numerical Results

[9] Equations (4) and (5) were solved using explicit and implicit finite difference techniques [e.g., *Fagherazzi et al.*, 2002]. The effect of piedmont diffusivity on mountain-belt denudation is illustrated in Figure 1b. In these two experiments, the model parameters are identical ($K = 10^{-7}$ yr^{-1}, L_m = 200 km, L = 500 km, and A_r = 5) except for the values of κ, which differ by a factor of 2. A smaller value of κ results in a steeper piedmont, lower bedrock relief, and a smaller denudation rate. After 250 Myr, the small-κ case has a mean elevation about twice as large as the large-κ case. Varying the piedmont length L for a fixed value of κ has a similar effect.

[10] Figure 1c illustrates the decline in mean drainage-basin elevation with time on a semi-log plot for a piedmont of fixed length (L = 500 km) and a range of values of K and κ. Following an initial rise to a maximum value, the decay in mean drainage-basin elevation is exponential in each case, but varies as a function of both the bedrock and piedmont parameters, illustrating their dual control on the tempo of denudation.

[11] In nature, values of K, κ, L_m, and L vary from one mountain belt to another. The purpose of this paper is not to constrain these values but to provide a model framework for determining how these parameters control the time scale of denudation. Persistent mountain belts can be expected to have extreme values of one or more of these parameters, resulting in large values of τ_d compared to the global average. L and L_m can be determined from topographic and geologic maps. Values of K and κ are not well constrained, but qualitative data on climate, rock type, and sediment texture can be used to vary those parameters around representative values.

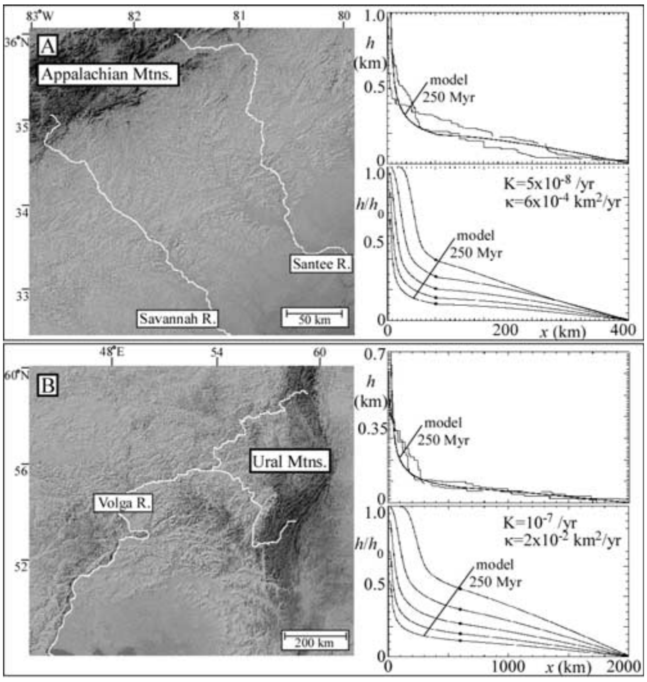

Figure 2. (A) Appalachian-type example. (Top right) Longitudinal profiles of channels in the Santee and Savannah drainage basins, plotted with a model profile ($K = 5 \times 10^{-8}$ yr^{-1}, $\kappa = 6 \times 10^{-4}$ km^2/yr) at $t = 250$ Myr for comparison. (Bottom right) Evolution of the model profile in intervals of 50 Myr. (B) Urals-type example. (Top right) Profiles in the Volga drainage basin, plotted with a model profile ($K = 10^{-7}$ yr^{-1}, $\kappa = 2 \times 10^{-2}$ km^2/yr) for comparison. See color version of this figure in the HTML.

[12] The effects of unusually resistant bedrock, for example, are illustrated in the Appalachian-type example of Figure 2a. *Hack* [1957, 1979] argued that the resistant quartzite of the Appalachian highlands was a key controlling factor in its evolution. In this example, the values of L_m and L were set to 100 km and 500 km. The values $K = 5 \times 10^{-8}$ yr^{-1} and $\kappa = 6 \times 10^{-4}$ km^2/yr led to profiles most similar to those observed (Figure 2a). In contrast, Figure 2b illustrates the Ural-type example (i.e., an unusually broad piedmont). Here, L_m and L were set to 600 km and 2000 km. The value of K most consistent with observed profiles in the Urals ($K = 10^{-7}$ yr^{-1}, $\kappa = 2 \times 10^{-2}$ km^2/yr; Figure 2b) is a factor of two greater than the value for the Appalachians. The Ural example shows that a broad piedmont can diminish bedrock relief, reducing the average drainage-basin denudation rate compared to a narrower piedmont of the same slope. The eastern Appalachian piedmont is relatively modest in length, so resistant bedrock is the most likely explanation for its persistence and strongly concave headwater profiles. The values of K and κ in Figure 2 are not unique; a range of values is consistent with the observed profiles.

4. Analytic Results

[13] The model equations can also be solved analytically for the decay phase of mountain-belt evolution using separation of variables. Power-law and sinusoidal solutions are obtained for the bedrock and alluvial components, and a transcendental equation is obtained for τ_d:

$$h_b(x,t) = h_0 \left(\frac{L_h}{x}\right)^{A_r/(K\tau_d)} e^{-t/\tau_d} \tag{8}$$

$$h_a(x,t) = h_0 \left(\frac{L_h}{L_m}\right)^{A_r/(K\tau_d)} \frac{\sin\left(\frac{L-x}{\sqrt{\kappa\tau_d}}\right)}{\sin\left(\frac{L-L_m}{\sqrt{\kappa\tau_d}}\right)} e^{-t/\tau_d} \tag{9}$$

$$\frac{\sqrt{\kappa\tau_b}}{L_m}\left(1 - \frac{A_r}{K\tau_d}\right) = \tan\left(\frac{L-L_m}{\sqrt{\kappa\tau_b}}\right) \tag{10}$$

Equation (10) cannot be further reduced and must be solved numerically for specific values of the model parameters. As an example, for the model parameters of the Appalachian-type case of Figure 2a, equation (10) gives $\tau_b = 186$ Myr. In the limit of no piedmont (i.e., $L_m \rightarrow L$), equation (10) has the appropriate limiting behavior $\tau_b \rightarrow A_r/K$.

[14] *Shepard* [1985] found actual bedrock profiles in nature to be best fit by a power law when slope was plotted versus downstream distance, consistent with equation (8). The power-law exponent in the model is a function of the bedrock erodibility (more resistant bedrock leads to more concave profiles) but the exponent is also a function of the foreland-basin diffusivity, κ (through τ_d), illustrating the explicit coupling of the bedrock and piedmont profile shapes. This result suggests that piedmont deposition has a direct influence on the morphology and denudation rates of upstream bedrock drainage basins.

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References

Ahnert, F. (1970), Functional relationships between denudation, relief, and uplift in large mid-latitude drainage basins, *Am. J. Sci.*, *208*, 243–263.
Baldwin, J. A., K. X. Whipple, and G. E. Tucker (2003), Implications of the shear stress river incision model for the timescale of postorogenic decay of topography, *J. Geophys. Res.*, *108*(B3), 2158, doi:10.1029/2001JB000550.
Braun, J., and M. Sambridge (1997), Modeling landscape evolution on geological time scales: A new method based on irregular spatial discretization, *Basin Res.*, *9*, 27–52.
Densmore, A. L., M. A. Ellis, and R. S. Anderson (1998), Landsliding and the evolution of normal-fault-bounded mountains, *J. Geophys. Res.*, *103*, 15,203–15,219.
Fagherazzi, S., A. D. Howard, and P. L. Wiberg (2002), An implicit finite difference method for drainage basin evolution, *Water Resour. Res.*, *38*(7), 1116, doi:10.1029/2001WR000721.
Fischer, K. M. (2002), Waning buoyancy in the crustal roots of old mountains, *Nature*, *417*, 933–936.
Hack, J. T. (1957), Studies of longitudinal stream profiles in Virginia and Maryland, *U.S. Geol. Surv. Prof. Pap.*, *294-B*, 42–97.
Hack, J. T. (1979), Rock control and tectonism: Their importance in shaping the Appalachian highlands, *U.S. Geol. Surv. Prof. Paper*, *1126-B*, 17 pp.
Howard, A. D. (1994), A detachment-limited model of drainage basin evolution, *Water Resour. Res.*, *30*, 2261–2285.
Humphrey, N. F., and P. L. Heller (1995), Natural oscillations in coupled geomorphic systems: An alternative origin for cyclic sedimentation, *Geology*, *23*, 499–502.
Kirchner, J. W., R. C. Finkel, C. R. Riebe, D. E. Granger, J. L. Clayton, J. G. King, and W. F. Megahan (2001), Mountain erosion over 10 yr, 10 k.y., and 10 m.y. time scales, *Geology*, *29*, 591–594.

Matmon, A., P. R. Bierman, J. Larsen, S. Southworth, M. Pavitch, and M. Caffee (2003), Temporally and spatially uniform rates of erosion in the southern Appalachian Great Smoky Mountains, *Geology*, *31*, 155–158.
Naeser, C. W., N. D. Naeser, M. J. Kunk, B. A. Morgan III, A. P. Schultz, C. S. Southworth, and R. E. Weems (2001), Paleozoic through Cenozoic uplift, erosion, stream capture, and deposition history in the Valley and Ridge, Blue Ridge, Piedmont, and Coastal Plain province of Tennessee, North Carolina, Virginia, Maryland, and District of Columbia, *Geol. Soc. Am. Abstr. Programs*, *33*, 312.
Paola, C., P. L. Heller, and C. L. Angevine (1992), The large-scale dynamics of grain-size variation in alluvial basins, 1. Theory, *Basin Res.*, *4*, 73–90.
Pinet, P., and M. Souriau (1988), Continental erosion and large-scale relief, *Tectonics*, *7*, 563–582.
Ruxton, B. P., and I. McDougall (1967), Denudation rates in northeast Papua New Guinea from potassium-argon dating of lavas, *Am. J. Sci.*, *265*, 545–561.
Shepard, R. G. (1985), Regression analysis of river profiles, *J. Geol.*, *93*, 377–384.
Snyder, N. P., K. X. Whipple, G. E. Tucker, and D. J. Merritts (2000), Landscape response to tectonic forcing: Digital elevation model analysis of stream profiles in the Mendocino triple junction region, northern California, *Geol. Soc. Am. Bull.*, *112*, 1250–1263.
Summerfield, M. A., and N. J. Hulton (1994), Natural controls on fluvial denudation rates in major world drainage basins, *J. Geophys. Res.*, *99*, 13,871–13,883.
Swenson, J. B., V. R. Voller, C. Paola, G. Parker, and J. G. Marr (2000), Fluvio-deltaic sedimentation: A generalized Stefan problem, *Eur. J. Appl. Math.*, *11*, 433–452.
Whipple, K. X., and G. E. Tucker (1999), Dynamics of the stream power river incision model: Implications for the height limits of mountain ranges, landscape response timescales, and research needs, *J. Geophys. Res.*, *104*, 17,661–17,674.
Willett, S. D. (1999), Orogeny and orography: The effect of erosion on the structure of mountain belts, *J. Geophys. Res.*, *104*, 28,957–28,981.
Zhang, P., P. Molnar, and W. R. Downs (2001), Increased sedimentation rates and grain sizes 2–4 Myr ago due to the influence of climate change on erosion rates, *Nature*, *410*, 891–897.

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