Statistical self-similarity of magmatism and volcanism

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Abstract. Magmatism and volcanism exhibit spatial and temporal clustering on a wide range of scales. Using the spatial pair-correlation function, the number of pairs of magmatic or volcanic events whose separation is between \( r - \frac{1}{2} \Delta r \) and \( r + \frac{1}{2} \Delta r \) per unit area, we quantify the spatial clustering of magmatism and volcanism in several data sets. Statistically self-similar clustering characterized by power law spatial pair-correlation functions is observed. The temporal pair-correlation function is used to identify self-similar temporal clustering of magmatism and volcanism in the Radiometric Age Data Bank of 11,986 dated intrusive and extrusive rocks in the North American Cordillera. The clustering of magmatism and volcanism in space and time in this data set is found to be statistically self-similar and identical to those of distributed seismicity. The frequency-size distributions of eruption volume and areal extent of basaltic flows are also found to be self-similar with power laws analogous to the Gutenberg-Richter distribution for earthquakes. In an attempt to understand the origin of statistical self-similarity in magmatism and volcanism we present one end-member model in which the ascent of magma through a disordered crust of variable macroscopic permeability is modeled with a cellular-automaton model to create a distribution of magma supply regions which erupt with equal probability per unit time. The model exhibits statistical self-similarity similar to that observed in the real data sets.

1. Introduction

Magmatism and volcanism are phenomena characterized by variable intensity on a wide range of spatial and temporal scales. For example, volcanic eruptions are episodic: like great earthquakes, they often occur without warning after a long period of quiescence. Such episodicity occurs over a broad range of timescales from historic timescales to much longer timescales recorded in deep-sea sediment cores as discussed by Kenneff et al. [1977], Kenneff [1981], and Kennett [1989]. They documented episodes of intense volcanism separated by long quiescent periods on the timescales of millions of years in Pacific Ocean sediment cores. Similarly, volcanism is localized in space. Magma erupts in volcanic zones with up to hundreds of individual volcanic vents separated by large distances with little or no volcanism [Guffanti and Weaver, 1988; Conner, 1990; Simkin, 1993]. The rate and spatial concentration of magmatic and volcanic activity depend on the scale of observation. For example, with respect to the Cenozoic magmatic history of the North American Cordillera, Armstrong and Ward [1991, p. 13203] observed that "although magmatism is always locally episodic, the episodes blur into a continuum of magmatic activity when the whole Cordilleran region is viewed during later Cenozoic time." Spatiotemporal clustering of volcanism is recognized in models in which the volcanic intensity of a region depends on the intensity of neighboring regions [Conner and Hill, 1995; Condit and Conner, 1996]. How magma supplied from a subducting slab upwells through the continental crust to erupt episodically at localized points is not well understood. The purpose of this paper is to quantify the variability of magmatic and volcanic intensity in space and time, including the frequency-size distribution and the spatiotemporal clustering of events, using a variety of techniques and data sets. In all cases, the measures of magmatism and volcanism are found to exhibit power law relationships indicative of statistical self-similarity. Self-similar behavior in space and time has previously been documented in hot spot volcanism by Shaw and Chouet [1991]. A model of fluid invasion in porous media has been introduced in the physics literature which gives rise to statistically self-similar behavior. We propose this model as one possible model of large-scale magma migration through a crust of variable macroscopic permeability for the formation of magma supply regions.

The organization of the text is as follows. First, evidence is presented that the cumulative frequency-size distribution of volcanic eruptions and basalt flows is a power law analogous to the Gutenberg-Richter law for earthquakes. Since there is evidence that the size of an eruption scales with the size of the magma supply re-
region that produced it, these results suggest that magma supply regions may also exhibit a power law cumulative frequency-size distribution.

The spatial clustering of volcanic vents and basaltic flows is then considered. We utilize the spatial pair-correlation function, the number of pairs of volcanic events whose separation is between $r - \frac{1}{2} \Delta r$ and $r + \frac{1}{2} \Delta r$ per unit area. We find that power laws are applicable to the spatial pair-correlation function for many data sets, indicating that the clustering is statistically self-similar.

As a model for the size distribution and clustering of volcanic events and magma supply regions, we utilize the observation that temperature variations in the uppermost mantle are self-affine. Self-affine functions are those which have a power spectrum with a power law dependence on wave number, $k$: $S(k) \propto k^{-\beta}$, where $\beta$ is a characteristic exponent. Self-affine temperature variations are inferred from the observation of self-affine regional variations in seismic wave speed at depth within Earth's mantle [Passier and Sneider, 1995]. In our model, magma supply regions are those where the isothermic depth is larger than a threshold value required for the melt fraction to be greater than the percolation threshold. Synthetic magma supply regions and distributions of volcanic rocks are constructed and their statistics compared to those from the western Great Basin. Close agreement is obtained between the statistics of real and model magmatic and volcanic regions.

Spatiotemporal clustering of magmatism in the North American Cordillera is considered with the Radiometric Age Data Bank of 11,986 radiometrically-dated intrusive and extrusive, basaltic and nonbasaltic igneous rocks. The large number of rocks in the database enables us to examine the clustering in time as a function of the distance separating rocks and, conversely, the clustering in space as a function of time. It is found that power law statistics are applicable to the temporal pair-correlation function but that the exponent of the temporal pair-correlation function characterizing the strength of the clustering depends on the spatial separation such that locations close together in space have highly correlated sequences of events in time. Similarly, the clustering in space is stronger for rocks separated by a small interval of time. The pair-correlation functions we compute with the Radiometric Age Data Bank are identical to those computed for distributed seismicity by Kagan and Knopoff [1980] and Kagan and Jackson [1991], indicating that distributed volcanism and distributed seismicity may have similar spatiotemporal dynamics.

Next, we discuss the dependence of rates of magma emplacement and volcanic output on the timescale over which the rate is computed. As with other episodic phenomena in Earth science such as sedimentation [Sadler, 1981; Sadler and Strauss, 1990; Strauss and Sadler, 1989] and morphological changes in species [Gingerich, 1983, 1993], the rate of magma emplacement and volcanic output is observed to be a decreasing power law function of the timescale of observation. This indicates that as the timescale is increased, more long periods of quiescence are included, reducing the rates of magmatism and volcanism. This relationship is another way to characterize the episodicity and intermittency of magmatism and volcanism.

Last, we introduce a model for the upwelling of magma through the continental crust that produces self-affine variations in temperature and a self-similar frequency-size distribution and spatiotemporal clustering. Our model considers the penetration of magma in a crust of variable macroscopic permeability. The variable permeability tends to roughen the isotherms characterizing the ascent, allowing magma to be advected more easily in regions of high permeability. Self-affine variations in isotherm depths are produced by this model. This is analogous to experiments in which fluids penetrating porous media develop interfaces that are self-affine [Buldyrev et al., 1992]. The magma supply regions formed in this model of upwelling magma are allowed to erupt with a random probability. As a result of the penetration of magma into new regions, magmatism and volcanism migrate over time in a realistic way in this model. Spatiotemporal data from a model of magmatic and volcanic processes are produced and the statistics are compared to those of the Radiometric Age Data Bank. Identical clustering statistics are obtained, suggesting that this model is one possible end-member model for magmatic and volcanic processes.

2. Frequency-Size Distribution of Volcanic Eruptions

The frequency-size distribution of volcanic events from a variety of geologic and marine environments is observed to be a power law analogous to the Gutenberg-Richter law for earthquakes. The Gutenberg-Richter law states that the number of earthquakes with a seismic moment greater than $M_o$ is a power law function with an exponent close to $-\frac{2}{3}$: $N(> M_o) \propto M_o^{-B}$, where $B \approx \frac{2}{3}$ [Kanamori and Anderson, 1975]. Simkin [1993] has compiled data on Holocene eruptions from small, monthly events to globally catastrophic events such as the 1815 eruption of Tambora and the eruption of the basalt flows of Yellowstone 2 Ma to obtain an approximate frequency-size distribution of volcanic eruptions. The volcanic explosivity index (VEI), defined to be the logarithm (base 10) of the erupted tephra volume, was used. Simkin [1993] found an inverse relationship between eruption volume and frequency of events with an exponent of approximately -1.

As further evidence for a power law frequency-size distribution in volcanism, we consider the distribution of basaltic rocks in the western Great Basin, mapped in Figure 1 of Yogodzinski et al. [1996]. We digitized the basaltic regions from the map west of 117° longitude.
and computed the cumulative frequency-size distribution of area (Figure 1), the number of areas larger than an area $A$. The result is plotted on logarithmic scales as the solid circles in Figure 2. The data from Figure 2 fit a power law distribution with an exponent of $-0.8$, $N(>A) \propto A^{-0.80 \pm 0.03}$, indicated by the solid line. The power law fit was obtained with a linear least squares regression to the logarithms (base 10) of the data. The same procedure was used to obtain all of the power law trends quoted in the analyses in this paper. The uncertainty in the exponent was obtained by dividing the population of basalt flows into two halves (each flow was randomly assigned to one group or the other) and calculating a least squares fit to the frequency-size distributions of each half of the data. The difference in the exponents between the two halves of the data is the estimate of the uncertainty. The uncertainties calculated in this way are comparable to standard errors obtained through the least squares procedure and provide an independent evaluation of the uncertainties of the exponents associated with the size of the data set.

This procedure for uncertainty estimation has been repeated for all of the uncertainties quoted in this paper.

3. Spatial Clustering of Volcanoes and Basaltic Flows

We have also considered the spatial clustering of the basaltic flows from the western Great Basin. In addition to the broad size distribution apparent in Figure 1 with many small regions and a few large ones, the basaltic regions are also clustered in space with large distances separating the zones of concentrated volcanism. We have utilized the spatial pair-correlation function to characterize the clustering. The spatial pair-correlation function $c(r)$ is the number of pairs of events whose separation is between $r - \frac{1}{2} \Delta r$ and $r + \frac{1}{2} \Delta r$ per unit area [e.g., Turcotte, 1997]. One point is picked, and the distances to all other points are determined. The same thing is done for the second point and for all other points. The number of pairs in each interval $\Delta t$ is divided by $\Delta t$ to obtain $c(t)$. For a one-dimensional distribution such as the temporal pair-correlation function that we will consider shortly, the number of pairs in each interval $\Delta t$ is divided by $\Delta t$ to obtain $c(t)$. The spatial pair-correlation function has been used by Kagan and Jackson [1991] to study earthquake clustering and by Pelletier and Turcotte [1996] to study the clustering of wells showing hydrocarbons in the Denver and Powder River basins. The point set in our analysis is the set of grid points in which basalt occurs. Alternatively, one could compute the centroid of each distinct basaltic region and use that point set.
to compute the spatial pair-correlation function. We performed both analyses and obtained similar results. The spatial pair-correlation function for the basaltic regions of Figure 1 is presented in Figure 3 on logarithmic scales. The data are well approximated by a power law function of the form $c(r) \propto r^{-0.85\pm0.05}$, indicating that the basaltic regions are clustered in a statistically self-similar way.

Before we consider other data sets for the spatial distribution of volcanic activity we will show how magma supply regions can be related to regions of high melting intensity. The power spectrum of regional variations in seismic wave speed of the upper mantle has been considered by Passier and Snieder [1995]. They computed variations in seismic wave speed at a given depth in Europe and the Mediterranean with techniques of seismic tomography and performed power spectral analyses on the resultant models. Variations in seismic wave speed are associated principally with variations in temperature [Hearn et al., 1991]. Passier and Snieder [1995] found seismic wave speeds to have a power law power spectrum with an exponent of approximately $-2$, $S(k) \propto k^{-2}$, from harmonic degree $l \approx 30$ to $l \approx 300$. Since this power spectrum is similar to that of a Brownian walk, we have used this result to construct a geometrical model of magma supply regions. The model is illustrated in Figure 4. In Figure 4 the Brownian walk represents a transect of variations in isothermic depth labeled as $T = T_c$. If the isothermic depth at any point along the transect is less than a critical value where the pressure $P = P_c$ is such that the melt fraction is greater than the percolation threshold, large-scale advection of magma will occur, forming a magma chamber and initiating active volcanism. Magma supply regions in the model are shaded in Figure 4. In this model, domains of a Brownian surface greater than a threshold value are associated with regions of active volcanism. The variations in isothermic depth are given by the Brownian walk, generated by successive coin flips, labeled $T = T_c$. If the isothermic depth is less than a critical value where the pressure $P = P_c$ is such that the melt fraction is greater than the percolation threshold, large-scale advection of magma will occur, forming a magma chamber and initiating active volcanism in that region.

We have created synthetic magma supply regions by constructing a Brownian walk surface ($\beta = 2$) using the Fourier-filtering technique described by Turcotte [1997]. Domains greater than a threshold value are shown in Figure 6 in map view. The threshold value was chosen such that the percentage of area magmatically active matched that of the percentage of area with basalts in the western Great Basin (6%). The size distribution of areas in Figure 6 is presented in Figure 7. The size distribution of areas is identical to that of basaltic regions in the Great Basin: $N(>A) \propto A^{-0.86\pm0.03}$. This result can also be shown theoretically. Kondev and Henley [1995] have related the length distribution of contour lengths of Gaussian surfaces to the Hausdorff measure $Ha$. The Hausdorff measure is related to the power spectral exponent $\beta$ by the relation $\beta = 2Ha + 1$ [e.g., Turcotte, 1997]. Kondev and Henley [1995] have given the size distribution of contour lengths (the probability that a randomly chosen contour loop has a length $s$) as $N(s) \propto s^{-\tau}$, where $\tau = 1 + (2 - Ha)/D$ and $D$ is the fractal dimension of the contours. The cumulative distribution (the number of contours with length greater than $s$) is the integral of the noncumulative distribution,
Figure 5. Illustration of the model relating variations in isothermic depth to the geometry of magma supply regions in three dimensions. (a) Shaded relief image of synthetic Brownian walk surface. (b) Shaded relief image of synthetic magma supply regions where the isothermic depth is less than a threshold value.

\[ N(>s) \propto s^{-(2-H_\alpha)/D} \]  

Since the length of a contour is related to the area it encloses by \( s \propto A^{D/2} \) (by definition), the cumulative distribution of areas enclosed by contours is \( N(>A) \propto A^{-(2-H_\alpha)/2} \). For a surface with \( \beta = 2, H_\alpha = 1/2 \), and we have \( N(>A) \propto A^{-3/4} \), nearly consistent with the size distribution of Figures 2 and 7. Since the Hausdorff measure of the interface is related to the exponent of the frequency-size distribution through a one-to-one function, the observed exponents are consistent with the Brownian walk variations of temperature at a given depth and the model for volcanic regions as extremal domains of a Brownian walk surface. Pelletier [1997] applied these calculations to the statistics of cumulus cloud fields and inferred from them the self-affine variability of the top of the convective boundary layer.

The similarity of the cumulative frequency-size distribution of synthetic magma supply regions and that of the basalt flows from the western United States suggests that the size of eruption volume and the size of the magma supply region that produced it may be strongly correlated. Smith [1979] has argued such a relationship for ash flow magmatism. Further similarity between the distribution of model magma supply regions and basaltic flows is given by the similarity of their spatial pair-correlation functions. The spatial pair-correlation function of the synthetic volcanic regions of Figure 6 is presented in Figure 8. The data fit a power law trend of the form \( c(r) \propto r^{-0.87} \), in close agreement with the pair-correlation function of the Great Basin basalts shown in Figure 3 which has a least squares regression to \( c(r) \propto r^{-0.85\pm0.04} \).

Figure 6. Distribution of model basaltic regions produced with the model of Figure 4. A Brownian walk surface with a power law power spectrum and \( \beta = 2 \) was generated with the Fourier-filtering method [e.g. Turcotte, 1997] to represent variations in isothermic depth in the upper mantle. Model magma supply regions are associated with domains of the Brownian walk surface with depth less than a threshold value. The threshold value was chosen such that the percentage of area as a supply region was equal to the percentage in Figure 1 (6%).

Figure 7. Cumulative frequency-size distribution of model basaltic regions \( N(>A) \) (solid circles), the number of regions with an area greater than \( A \), for the distribution in Figure 6. A least squares fit to the logarithms of the data yield the relationship \( N(>A) \propto A^{-0.80} \) shown by the straight line. This distribution is identical to that of basalts from the western Great Basin.
We have also calculated the spatial pair-correlation function for two other published distributions of volcanic activity. In Figure 9 we present the pair-correlation function of volcanic events from four time intervals of the history of the Springerville volcanic field in Arizona digitized from Figure 9 of Condit and Conner [1996]. The pair-correlation functions from the oldest time interval to the most recent are presented as solid circles, open circles, crosses, and squares in Figure 9, respectively. Activity in the Springerville volcanic field over time from 1.75 to 0.75 Ma in 0.25 Myr increments as illustrated in Figure 9 of Condit and Conner [1996] is a particularly striking example of migration of volcanic activity from one region to another. Despite the fact that the loci of activity are quite different from one time increment to another the pair-correlation function for each time increment is nearly identical, suggesting that statistically self-similar clustering is not a transient feature but a fundamental characteristic of volcanism. The observation of statistical self-similar clustering of hydrothermal veins at scales of centimeters to meters [Manning, 1994; Magde et al., 1995] suggests that self-similar clustering holds on more than one range of spatial scales.

The second data set is cinder cones from the Mauna Kea volcano, Hawaii, mapped by Porter [1972]. The pair-correlation function of this distribution is presented in Figure 10. Statistically self-similar clustering is observed as with the previous data sets. The exponent, however, is -0.50±0.05, smaller in magnitude than the exponents previously obtained.

The conclusions we have reached on clustering of volcanism are inconsistent with those of de Buemond d'Ars et al. [1995]. They considered the spatial distribution of volcanoes in active margins and have concluded, based on an analysis of the distribution of nearest-neighbor spacings between volcanoes, that volcanoes were randomly distributed with no clustering. The difference in our conclusions can most likely be ascribed to the different methods we employ. The pair-correlation function uses data for the spacing between every pair of volcanoes, not just those between nearest neighbors, and therefore gives a complete representation of the original distribution while nearest-neighbor distances maintain only a small subset of the information contained in the data set (N versus N(N - 1)/2, where N is the number of points).
4. Temporal Clustering of Igneous and Eruption Events

In this section we consider the Radiometric Age Data Bank, a collection of locations and ages of 11,986 dated igneous rocks from the North American Cordillera [Zartman et al., 1976; Armstrong and Ward, 1991; Ward, 1995]. Since the database consists of both basaltic and nonbasaltic rocks, the analysis of clustering in this data set yields additional information than the clustering of strictly basaltic rocks in Figure 1. The spatial distribution of rocks in this database, obtained from P. Ward (personal communication, 1996), is shown in Figure 11. A representation of the data which illustrates the variability of magmatism through time is presented in Figure 12. Figure 12 plots the distribution of igneous rocks in time against the position projected along the plate margin from Cape Mendocino to Guadalupe and suggests that both spatial and temporal clustering occur in this data set.

There are two potential biases in the sampling of igneous rocks: proximity of the sample locations to roads and differential erosion and/or deposition which may remove or bury rocks in certain locations more than in others. If, for example, there is a higher frequency of sampling in areas closer to roads the Radiometric Age Data Bank may not be a uniform record of magmatism and volcanism in the North American Cordillera. We have ruled out the possibility of significant sampling bias due to the proximity of roads by computing the cumulative probability distribution of population density for all land areas in the 15øx15ø square from 120øW to 105øW longitude and 35øN to 50øN latitude. This distribution can be compared to that for only those land areas where rocks are sampled. Population density data were obtained from the Global Demography Project [Tobler et al., 1995]. The two distribution functions are presented in Figure 13a with the distribution for all land areas as the solid line, and the distribution for only land areas where rocks are sampled as the dashed line. The two distributions have nearly the same shape. However, since the dashed line lies above the solid line, the frequency of sampled rock in areas with a population density of at least 1 person/km² is slightly higher than the frequency of all such land areas. There are two possible reasons for this. One is that rock sampling is slightly biased toward more populated areas. Another possibility is that the igneous rocks happen to be clustered in regions with, on average, slightly higher population density. This is likely since population density is tightly clustered with the extremely depopulated areas of the study area concentrated in the basin and range. Two clustered distributions may not overlap even if they are independent of one another. Nevertheless, we conclude from the close similarity of the distributions in Figure 13a that population density represents at most a small bias in the sampling of rocks in the Radiometric Age Data Bank.

To test for possible sampling bias due to differential erosion and/or deposition, we have computed the cumulative probability distribution function as a function of elevation. Relief alone is an accurate predictor of denudation rates [Ahnert, 1970; Pinet and Souriau, 1988]. We have taken advantage of this correlation to use elevation as a proxy for erosion and deposition. The two distributions, analogous to those for population density, are presented in Figure 13b. The elevation data used were extracted from the ETOP05 digital elevation model [Loughridge, 1986]. The close correspondence of the two distributions indicates that no denudational sampling bias exists in the data set.

The large number of rocks in the database enables us to quantify the clustering in space for different ranges.
of age separation between the rocks and, conversely, to quantify the clustering in time for different spatial separations. The temporal pair-correlation function in time is presented in Figure 14 for radial separations $r = 0.3-1$ km, 1-3 km, 3-10 km, 10-30 km, 30-100 km, and 100-300 km, presented from top to bottom, respectively. The temporal pair-correlation function has also been used by Pelletier [1999] to characterize the clustering of geomagnetic reversals. The strongest clustering, characterized by $c(t) \propto t^{-1}$, is observed for pairs of rocks separated by a small distance. The clustering weakens, i.e., the pair-correlation function is flatter, as the radial separation increases. The same analysis has been performed on earthquake catalogs by Kagan and Jackson [1991] with strikingly similar results. They observed the pair-correlation function to depend on time as $c(t) \propto t^{-1}$ for earthquake pairs with small separations. A progressively weaker, power law dependence was observed for larger separations. This is consistent with the results for the pair-correlation analyses of Kagan and Jackson [1991] for small spatial separations. Identical clustering statistics for distributed volcanism and distributed seismicity suggest a similar dynamics for the two phenomena. Clustering of distributed seismicity is related to strain weakening on faults [Dieterich, 1992]. We point out the similarity in spatiotemporal clustering of distributed seismicity and magmatism and volcanism in the hope that understanding of one phenomenon may be brought to bear on the other even though they are very different physical processes.

The complementary analysis of the spatial pair-correlation function for rocks separated by different time ranges is presented in Figure 15. The time ranges corresponding to the different curves are, from top to bottom, $t = 0.03-0.1$ Myr, 0.1-0.3 Myr, 0.3-1 Myr, 1-3 Myr, 3-10 Myr, and 10-30 Myr, respectively. The same behavior is obtained for $c(r)$ as for $c(t)$: the clustering for the smallest time separations is given by $c(r) \propto r^{-1}$, with progressively weaker clustering characterizing rocks with greater age differences.
Figure 15. Spatial pair-correlation function in space of igneous rocks from the North American Cordillera compiled in the Radiometric Age Data Bank for time ranges $t = 0.03-0.1$ Myr, $0.1-0.3$ Myr, $0.3-1$ Myr, $1-3$ Myr, $3-10$ Myr, and $10-30$ Myr, from top to bottom, respectively. The same behavior is obtained for $c(r)$ as for $c(t)$: the clustering for the smallest time separations is given by $c(r) \propto r^{-1}$, with progressively weaker clustering characterizing rocks with greater age differences.

5. Scaling of Volcanic Activity Rate Versus Time

The observation of similar clustering behavior in space and time suggests that temporal variations in volcanic flux at a single location may have statistically self-similar behavior just as spatial variations in volcanic flux at a single instant of time are statistically self-similar. For instance, consider Figure 4 to represent variations in the isothermic depth at a single location as a function of time rather than variations in isothermic depth along a transect at an instant of time. The shaded regions now represent periods of volcanic activity which can turn on and switch off through time. A plot of the cumulative volcanic flux $Q_{\text{cum}}$ produced with this model is presented in Figure 16. To construct Figure 16 we have generated a fractional Brownian walk with the Fourier-filtering method of Turcotte [1997]. A fractional Brownian walk is a time series with a power law power spectrum with $1 < \beta < 2$ [Turcotte, 1997]. We have constructed the time series with $\beta = 1.8$, slightly smaller than 2, since the model considered in section 6 produces temporal variations with $\beta = 1.8$. An eruption occurs when the isothermic depth is above a critical value. Eruptions occur instantaneously, releasing a flux of magma equal to the area of the shaded region. Eruptions can be identified by vertical lines in Figure 16. When the isothermic depth is below the critical value, no volcanism occurs and a period of quiescence, indicated by a horizontal line of constant cumulative volcanic output in time, is observed. One way to characterize the episodicity of magmatic output produced by this model is to calculate the volcanic flux as a function of the timescale of observation.

We will next discuss the relationship between magmatic output and time span based on a model with a deterministic fractal distribution of quiescence periods. We consider a vertical sequence of volcanic beds. The age of volcanic sequences in this model is given as a function of depth in Figure 17a. As illustrated, the vertical segments (beds) are of equal thickness. The positions of the transitions from beds to hiatuses are given by a second-order Cantor set. Eight kilometers of volcanic rock have been deposited in this model sequence in a period of 9 Myr so that the mean rate of volcanism is $R(9 \text{ Myr}) = 8 \text{ km}/9 \text{ Myr} = 0.89 \text{ mm/yr}$ over this period. However, there is a major unconformity at a depth of 4 km. The rocks immediately above this unconformity have an age of 3 Ma and the rocks immediately below it have an age of 6 Ma. There are no rocks in the volcanic sequence with ages between 3 and 6 Ma. In terms of the Cantor set this is illustrated in Figure 17b. The line of unit length is divided into three parts, and the middle third, representing the period without volcanism, is removed. The two remaining parts are placed on top of each other as shown.

During the first 3 Myr of eruptions (the lower half of the volcanic sequence) the mean rates of volcanism are $R(3 \text{ Myr}) = 4 \text{ km}/3 \text{ Myr} = 1.33 \text{ mm/yr}$. Thus the rate of volcanism increases as the period considered decreases. This is shown in Figure 17c.

There is also an unconformity at a depth of 2 km. The rocks immediately above this unconformity have an age of 1 Ma, and rocks below have an age of 2 Ma. Similarly, there is an unconformity at a depth of 6 km, the rocks above this unconformity have an age of 7 Ma, and rocks below have an age of 8 Ma. There are no rocks in the pile with ages between 8 and 7 Ma or between 2 and 1.
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Figure 17. Illustration of a model for volcanic sequences based on a Devil's staircase associated with a second-order Cantor set. (a) Age of volcanic rocks \( T \) as a function of depth \( y \). (b) Illustration of how the Cantor set is used to construct the volcanic sequence. (c) Average volcanic flux \( R \) as a function of the period \( T \) considered.

This is clearly illustrated in Figure 17a. In terms of the Cantor set (Figure 17b) the two remaining line segments of length 1/3 are each divided into three parts and the middle thirds are removed. The four remaining segments of length 1/9 are placed on top of each other as shown. During the periods 9 to 8, 7 to 6, 3 to 2, and 1 to 0 Myr the rates of volcanism are \( R (1 \text{ Myr}) = 2 \text{ km/1 Myr} = 2 \text{ mm/yr} \). This rate is also included in Figure 17c.

The volcanic flux clearly has a power law dependence with respect to the length of the time interval considered. The results illustrated in Figure 17 are based on a second-order Cantor set but the construction can be extended to any order desired and the power law results given in Figure 17c would be extended to shorter and shorter time intervals.

One thousand samples of volcanic flux calculated between pairs of randomly chosen instants in Figure 16 are plotted in Figure 18. The volcanic flux has a power law dependence on time span with exponent \(-0.6\): \( R \propto T^{-0.60_{-0.03}^{+0.02}} \). The decrease in rate results from the appearance of longer hiatuses in longer time intervals is illustrated in Figure 16. These results can also be obtained from theoretical fractal relations. Fractional Brownian walks have the property that the standard deviation of the time series has a power law dependence on time with a fractional exponent \( H_a \), the Hausdorff measure: \( \sigma \propto T^{H_a} \). The rate of change of the time series for a given time interval \( T \) is then the volcanic flux \( R = \sigma /T \propto T^{H_a-1} \). For \( \beta = 1.8, H_a = 0.4 \), and the volcanic flux is then \( R \propto T^{-0.60} \), in agreement with the numerical result in Figure 18.

The results of this model can be compared to published volcanic fluxes. Crisp [1984] has compiled estimated rates of magmatic output along with the area and time over which the output occurred. We have estimated the volcanic flux as the volcanic rate estimated by Crisp [1984] divided by the area of the region. The volcanic flux for active margins estimated in this way is plotted as a function of the timescale of observation in Figure 19. Since the data are collected from many regions in the world, significant scatter in the data set is observed. However, fluxes have a clear inverse dependence on timescale. A least squares fit to the data yields the relationship \( R \propto T^{-0.65_{-0.07}^{+0.05}} \), indicated by

Figure 18. Average volcanic flux \( R \) as as function of time span \( T \) for the volcanic sequence of Figure 16.

Figure 19. Estimated volcanic and magmatic fluxes as a function of the timescale over which the flux is estimated from the data set of Crisp [1984]. Volcanic flux has an inverse dependence on timescale with a least squares fit to \( R \propto T^{-0.65} \).
the solid line, in close agreement with the model result $R \propto T^{-0.60}$ described above. On historical timescales, fractal behavior characterized by a power law distribution of quiescence periods in the histories of basaltic volcanoes has been identified by Dubois and Cheminee [1991].

6. Self-affine Model of Magmatic Upwelling Through the Continental Crust

Experiments and models of fluid migration in a disordered porous medium have been considered by Buldyrev et al. [1992]. In their experiments, fluid was injected into one end of a sponge-like material called oasis and commonly used by florists. After some time had passed, the material was dissected, and the interface between the injected fluid and the displaced air was scanned. The final interface was rough: the distance of the interface from the injection point was highly variable, despite the fact that the medium was macroscopically uniform. The reason for the rough interface is that capillary forces roughen the interface by advecting the fluid into the pore spaces. As the fluid penetrates into the material, it encounters more pore spaces which further roughen the surface creating a macroscopically variable interface over time. Surface tension tends to smooth the interface. The result is a dynamic steady state between these two competing effects. The statistics of the interface were studied in this experiment and were found to be characteristic of a Brownian walk with $S(k) \propto k^{-2}$.

A cellular-automaton model for this phenomenon was considered by Buldyrev et al. [1992]. The rules of the model are illustrated for a two-dimensional cross section in Figure 20. On a cubic lattice a fraction $p$ of cells are "blocked" to represent zones of low macroscopic permeability into which the fluid cannot easily penetrate. "Unblocked" cells are those in which the fluid can penetrate more easily. A portion of a transect of the model at $t = 0$, where the interface is a flat surface at its maximum depth, is shown in Figure 20a. The blocked sites are denoted by circles and the unblocked sites are denoted by vertical lines. At $t = 1$ a cell (labeled X in Figure 20b) is randomly chosen from among the unblocked cells adjacent to the interface. Fluid is advected into cell X, and any cells that are below it in the same column. This process is then iterated. For example, Figure 20c shows that at $t = 2$ we choose a second unblocked cell, cell Y, to fill, while Figure 20d shows that at $t = 3$ we fill cell Z and also cell Z' below it. With this rule, blocked sites are eventually filled, but it takes longer to penetrate them in accordance with their lower permeability. This model produces self-affine behavior in both space and time. Variations in interfacial depth along a transect have a power spectrum $S(k) \propto k^{-2}$, and variations in time at a single point have a power spectrum $S(f) \propto f^{-1.8}$ [Buldyrev et al., 1992; Havlin et al., 1995].

We have combined the model of Buldyrev et al. [1992] with eruption dynamics as a model for the upwelling of magma through the disordered crust to obtain a model history of volcanism in space and time. Isotherms evolve according to the dynamics of the model of Buldyrev et al. [1992]. The probability $p$ was chosen to be 0.4, slightly below the critical probability $p_c \approx 0.47$ where the interface will encounter a connected sequence of blocked sites. At each time step, potentially eruptive magma chambers are identified as domains with an isothermic depth less than a threshold value as in Figure 5. During that time step, each magma chamber has an equal probability to erupt. Since the magma chambers have a power law frequency-size distribution, an equal failure probability per unit time for each magma chamber implies a power law cumulative frequency-size distribution of the form $N(> V) \propto V^{-0.8}$ consistent with the observations of a frequency-size distribution of eruptions. As the isotherms ascend in the model, more cells are added at the top so that the model maintains a long-term average crustal density in the model. Magma chambers refill immediately after eruption.

In Figure 21 we present a map of erupted magma supply regions from two consecutive time intervals of equal duration. The regions that erupt during the first time slice are shown shaded in grey, while those from the second time slice are shown in black. Figure 21 illustrates the migration of volcanism in the model. The volcanism from the second time slice correlates strongly with
One popular model for generating spatial patterns of volcanism in active margins is the buoyancy instability [Marsh, 1982; Kerr and Lister, 1988; Whitehead and Helfrich, 1991]. Depending on the initial conditions and assumptions used, this model can generate periodic [Whitehead and Helfrich, 1991], random [de Buemond et al., 1995], or clustered [Kelly and Bercovici, 1997] distributions. In contrast, the effect of variable macroscopic permeability acts only to cluster, consistent with the empirical observations we have documented. We must stress, however, that the model described in this section is only one possible end-member model for magmatic ascension through the crust.

7. Conclusions

The purpose of this paper was to show that complex variability in space and time in magmatism and volcanism is characterized by robust self-similarity. Statistical self-similarity arises in four ways: (1) the frequency-size distribution of eruptions and basaltic regions, (2) the clustering of events in space, (3) the clustering of events in time, and (4) the relationship of volcanic flux with the timescale of observation. We have presented a model of magma migration through a disordered crust of variable macroscopic permeability which generates self-affine variations in isothermic depth in space and time. Extreme values of these variations in space and time can be associated with active regions and eruptions, respectively, and the statistics of those regions related to the self-affinity of the isothermic depth in a variety of ways. The model is, however, only one possible model consistent with the data.
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References


Sadler, P.M., Sediment accumulation rates and the completeness of stratigraphic sections, *J. Geol.*, 89, 569-584, 1981.


Ward, P.L., Subduction cycles under western North America during the Mesozoic and Cenozoic eras, in *Magmatism


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