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VARIATIONS IN SOLAR LUMINOSITY FROM TIMESCALES OF MINUTES TO MONTHS

JON D. PELLETIER[1]

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ABSTRACT

We present the power spectrum of solar irradiance during 1985 and 1987, obtained from the active cavity radiometer irradiance monitor project from timescales of minutes to months. At low frequency, the spectra are Lorentzian [proportional to $1/(f^2 + f_0^2)$]. At higher frequencies, they are proportional to $f^{-1/2}$. A linear, stochastic model of the turbulent heat transfer between the granulation layer (modeled as a homogeneous thin layer with a radiative boundary condition) and the rest of the convection zone (modeled as a homogeneous thick layer with thermal and diffusion constants appropriate the lower convection zone) explains the observed spectrum.

Subject headings: convection — diffusion — Sun: activity — turbulence

1. INTRODUCTION

The luminosity of the Sun has significant variations on timescales from minutes to years. Data from the *NIMBUS 7* and active cavity radiometer irradiance monitor (ACRIM) projects have provided us with a high-quality time series of these variations. Some aspects of these variations can be attributed to specific physical processes (see Stix 1989, for an introductory review). For example, oscillations of the Sun result in a well-understood 5 minute periodicity. At the yearly and decadal timescale, there is a strong correlation between the irradiance and variations in the solar magnetic activity. Variations from minutes to months have a simple spectral form but are not well understood (Frohlich 1993).

Frohlich (1987) has published power spectra of ACRIM data from 1980 and 1985. He reported that the power spectrum is flat for frequencies corresponding to timescales greater than 1 week, proportional to f^{-2} for timescales between 12 hr and 1 week, and proportional to f^{-1} at timescales down to minutes. Our studies agree that the low-frequency spectrum is Lorentzian. We find, however, that the high-frequency spectrum is proportional to $f^{-1/2}$ at timescales shorter than 1 day.

Kuhn, Libbrecht, & Dicke (1988) have suggested that the interaction between the solar surface and deeper portions of the convection zone may cause the low-frequency variations. We explore that possibility in this Letter.

The simplest way to model the convective transport of heat is to assume that the flux of heat is proportional to its gradient. Fluctuations in heat energy will then be governed by the diffusion equation. This is a valid approximation if the turbulence is small scale (dominated by eddies whose length scale is much smaller than that of the mean gradient of potential temperature; Garratt 1992). Because of the stochastic nature of turbulence, a stochastic diffusion model is appropriate for transport with the flux-gradient approximation. In this Letter, we study the fluctuations in luminosity of a thin homogeneous surface granulation layer with a radiation boundary condition, exchanging heat with a deep, homogeneous layer below, with density and diffusion constants appropriate to the lower convection zone. The high- and low-crossover frequencies in the spectrum correspond to timescales of thermal and radiative equilibration of the convection zone, respectively. The timescales for equilibration are given by the model as a function of thermal and diffusion constants of the granulation layer and lower convection zone. Estimates of these constants obtained from mixing length theory yield order-of-magnitude agreement between the crossover frequencies predicted by the model and those observed.

2. POWER SPECTRUM OF ACRIM DATA

In Figure 1, we present the logarithm (base 10) of the normalized Lomb periodograms of ACRIM solar irradiance data sampled during 1987 and 1985, plotted as a function of the logarithm of the frequency. We chose to analyze these years since they appear to represent extremes in the variation of solar activity at low frequencies. The low-frequency variances in the 1985 and 1987 data are small and large, respectively. Variations in the solar irradiance at yearly timescales are generally agreed to be the result of variations in magnetic activity. We have chosen these years in order to assess the influence of magnetic activity on the power spectrum and distinguish its influence from that of the mechanism proposed in our model.

Since the data were sampled at irregular intervals, simple fast Fourier transform (FFT) methods of estimating the power spectrum are only available if we average the data over some uniform time interval. We chose instead to use the Lomb periodogram suggested by Press et al. (1992) for unevenly sampled data. Above frequencies of $\log f = -2.0$, we averaged the periodogram in logarithmically spaced frequency intervals of $\log f = 0.01$, in order to reduce the scatter. We subtracted $\log S(f)$ by 2.5 to plot it on the same graph as the 1987 data.

The high-frequency behavior is the same for both spectra. An f^{-2} region flattens out to $f^{-1/2}$ at frequencies greater than $f \approx 1/(1\ \text{day})$. Our observation of a $f^{-1/2}$ scaling region at high frequencies disagrees with Frohlich's (1987) conclusion that the high-frequency scaling region is proportional to f^{-1}. He reported this conclusion for 1985 ACRIM data, the same data we analyzed

[1] Department of Geological Sciences, Snee Hall, Cornell University, Ithaca, NY 14853.

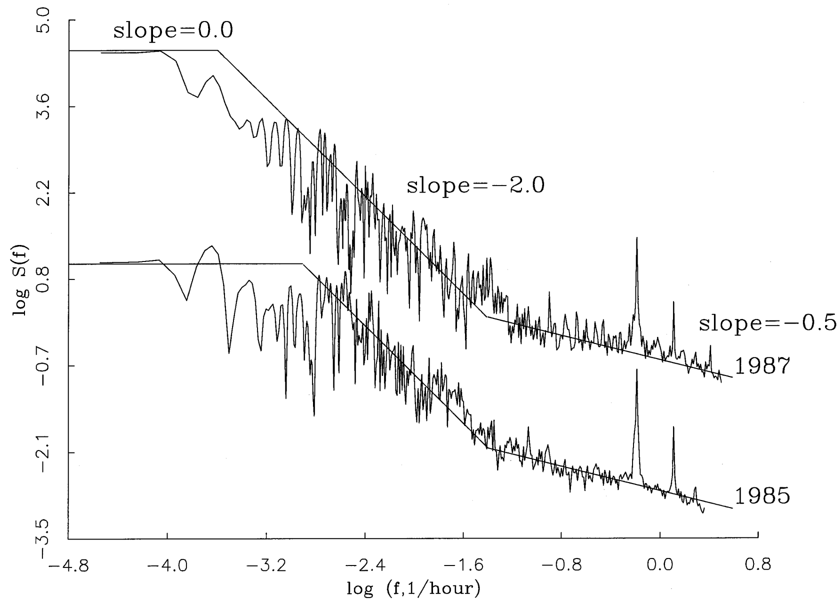

FIG. 1.—Logarithm (base 10) of the normalized Lomb periodograms of solar irradiance in 1987 (*upper plot*) and 1985 (*lower plot*) from the ACRIM project vs. the logarithm of the frequency in hr^{-1}. Crossover frequencies for 1987 are $f_0 = 1/(5 \text{ months})$ and $f_1 = 1/(1 \text{ day})$. The crossover frequencies for 1985 are $f_0 = 1/(1 \text{ month})$ and $f_1 = 1/(1 \text{ day})$. The 1985 spectrum is shifted down by $\log S(f) = 2.5$.

as part of our study. Our confidence in our interpretation lies in the greater resolution of our spectra, the $f^{-1/2}$ range of which is 50% larger than Frohlich's (1987). Large peaks appear at the orbital frequency of the satellite and its harmonics. These peaks are an artifact of the spectral estimation.

The low-frequency behavior of both spectra is Lorentzian (constant at low frequencies and proportional to f^{-2} at higher frequencies), in agreement with Frohlich's (1987) results. Aside from the basic form of the spectra, there is a large variability between the crossover frequencies and the magnitude of the two spectra reported here and those published by Frohlich (1987). This variability was also discussed by Frohlich (1987). The crossover frequencies of the Lorentzian portion of the 1987 and 1985 data reported here are $f = 1/(5 \text{ months})$ and $f = 1/(1 \text{ month})$, respectively. We interpret this variability as being due to either variations in the magnetic activity or to limitations of our model at these timescales.

3. MODEL OF VARIATIONS IN SOLAR LUMINOSITY

The variations in the irradiance of the Sun will be proportional to the variations in its surface temperature. This follows from the fact that the power emitted by the Sun (modeled as a blackbody), $F - F_e = \sigma T^4 - \sigma T_e^4$, can be well approximated by a linear dependence on $T - T_e$ for small departures from equilibrium.

Turbulent transport of heat in the convection zone of the Sun can be modeled by a stochastic diffusion process within the flux-gradient approximation. A stochastic diffusion process can be studied analytically by adding a noise term to the flux of a deterministic diffusion equation (van Kampen 1981):

$$\rho c \frac{\partial \Delta T}{\partial t} = -\frac{\partial J}{\partial x}, \tag{1}$$

$$J = -\sigma \frac{\partial \Delta T}{\partial x} + \eta(x, t), \tag{2}$$

where ΔT are the fluctuations in temperature from equilibrium and the mean and variance of the noise are given by

$$\langle \eta(x, t) \rangle = 0, \tag{3}$$

$$\langle \eta(x, t)\eta(x', t') \rangle \propto \sigma(x)\langle T(x) \rangle^2 \delta(x - x')\delta(t - t'). \tag{4}$$

To show how a power spectrum proportional to $f^{-1/2}$ can arise from a stochastic diffusion process, we will calculate the power spectrum of temperature fluctuations in a layer of width $2l$ exchanging heat with an infinite, one-dimensional, homogeneous space. Our derivation is nearly the same as that of Voss & Clarke (1976). Defining the Fourier transform as

$$J(k, \omega) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dt\, e^{-ikx} e^{i\omega t} J(x, t), \tag{5}$$

the Fourier transform of the heat flux of the stochastic diffusion equation is

$$J(k, \omega) = \frac{i\omega\eta(k, \omega)}{Dk^2 - i\omega}. \tag{6}$$

The rate of change of heat energy in the layer will be given by the difference in heat flux out of the boundaries, located at ± 1: $dE(t)/dt = J(l, t) - J(-l, t)$. The Fourier transform of $E(t)$ is then

$$E(\omega) = \frac{1}{(2\pi)^{1/2}}\,\omega \int_{-\infty}^{\infty} dk\, \sin(kl)\, J(k, \omega). \tag{7}$$

The power spectrum of variations in $E(t)$, $S_E(\omega) = |E(\omega)|^2$ is

$$S_E(\omega) \propto \int_{-\infty}^{\infty} \frac{dk\, \sin^2(kl)}{D^2k^4 + \omega^2} \propto \omega^{-1/2} \tag{8}$$

for low frequencies. Since $\Delta T \propto \Delta E$, $S_T(\omega) \propto \omega^{-1/2}$, as well.

At lower frequencies, the entire convection zone achieves thermal equilibrium and the lower convection zone can no longer absorb fluctuations in the heat flux from the radiation boundary condition. The fluctuating heat flux near the surface adds and subtracts heat from the top of the convection zone through the radiation boundary condition. This results in temperature and irradiance variations with a random-walk (f^{-2}) spectrum.

The fluctuating input and output of heat in the f^{-2} region will cause large variations from equilibrium. When the temperature of the convection zone becomes larger than the equilibrium temperature, it will radiate, on average, more heat than at equilibrium. Conversely, when the temperature of the convection zone wanders lower than the equilibrium temperature, less heat is radiated. This negative feedback limits the variance at low frequencies, resulting in a constant power spectrum.

The model we present was solved in the context of a different problem by van Vliet, van der Ziel, & Schmidt (1980). They considered the temperature fluctuations in a thin metal film supported by a substrate.

The geometry of the model is a thin granulation layer of width 2×10^6 m and uniform density (equal to the density at the bottom of the granulation layer, where most of the heat capacity resides) of 0.003 kg m^{-3} coupled to a thick layer of uniform density representing the rest of the convection zone. For convenience, the layers have a planar geometry in this simplified model. The turbulent diffusivity is estimated from mixing length theory to be $\alpha = \frac{1}{3}vl$, where v and l are the characteristic velocity and eddy sizes, respectively. The eddy size, l, is usually approximated as 1 pressure scale height. In the case of the granulation layer, however, the dominant eddy size is the size of the convection cell, approximately 2×10^6 m. The velocity at the bottom of the granulation layer is on the order of 1000 m s^{-1}. These estimates yield an eddy diffusivity of 10^9 m^2 s^{-1} near the solar surface. The thermal conductivity, $\sigma = \alpha\rho c$, is 3×10^7 W m^{-1} K^{-1} since $c = 10$ J kg^{-1} K^{-1} for a monatomic hydrogen gas. The velocity, thickness, and specific heat values are from a standard solar model presented in Stix (1992). The density values are from Bohm (1963).

The granulation layer sits atop the rest of the convection zone. We approximate the density, width, and diffusivity of the remainder of the convection zone by their values near the bottom of the convection zone because its high density results in a concentration of heat capacity there and its slow diffusivity is the rate-limiting step of thermal equilibration of the convection zone. The width and dominant eddy scale will both be given by 10^7 m, the width of the lowest pressure scale height of the convection zone. The density is estimated to be 0.5 kg m^{-3}, and the velocity is on the order of 20 m s^{-1}. These values are the arithmetic means of the densities and velocities at the top and bottom of the pressure scale height. These values yield an eddy diffusivity of 10^8 m^2 s^{-1} for the bottom of the convection zone. Our diffusivities agree with the estimates of Stix (1992), who quoted the range of diffusivities in the convection zone as 10^8–10^9 m^2 s^{-1}. The thermal conductivity is 3×10^8 W m^{-1} K^{-1}.

The equation for temperature fluctuations in space and time in the model is

$$\frac{\partial \Delta T(x, t)}{\partial t} - \alpha(x)\frac{\partial^2 \Delta T(x, t)}{\partial x^2} = -\frac{\partial \eta(x, t)}{\partial x}, \tag{9}$$

with

$$\langle \eta(x, t) \rangle = 0, \tag{10}$$

$$\langle \eta(x, t)\eta(x', t') \rangle \propto \sigma(x)\langle T(x) \rangle^2 \delta(x - x')\delta(t - t'). \tag{11}$$

The boundary conditions are that there be no heat flow out of the bottom of the convection zone and that continuity of temperature and heat flux at the boundary separating the granulation layer and the deeper convection zone be given by

$$\sigma' \left.\frac{\partial T}{\partial x}\right|_{x = w2} = 0, \tag{12}$$

$$\Delta T(x = w_1^+) = \Delta T(x = w_1^-), \tag{13}$$

$$\sigma \left.\frac{\partial \Delta T}{\partial x}\right|_{x = w_1^-} = \sigma' \left.\frac{\partial \Delta T}{\partial x}\right|_{x = w_1^+}, \tag{14}$$

where w_1 and w_2 are the widths of the granulation layer and deep convection zone, respectively, and the primes denote the thermal and diffusion constants of the deep convection zone.

At the top of the granulation layer, we impose a blackbody radiation boundary condition linearized about equilibrium,

$$\sigma \frac{\partial \Delta T}{\partial x}\bigg|_{x=0} = g\Delta T(x=0)\,, \tag{15}$$

where $g = 4\sigma_B T_0^3 = 2 \times 10^4$ W m^{-2} K^{-1} is the thermal conductance of heat out of the Sun and σ_B is the Stefan-Boltzmann constant.

Van Vliet et al. (1980) used Green's functions to solve this model. The power spectrum of the average temperature in the granulation layer, and hence the irradiance, is, using their solution,

$$S(f) \propto \mathrm{Re}\left[\left[L^2\left\{\frac{\sigma' L}{\sigma L'}\tanh\left(\frac{w_2}{L}\right)\left[\left(\frac{gw_1}{\sigma}-1\right)\tanh\left(\frac{w_1}{L}\right)-\frac{2gL}{\sigma}\frac{\cosh(w_1/L)-1}{\cosh(w_1/L)}+\frac{w_1}{L}\right]\right.\right.\right.$$
$$\left.\left.\left.+\frac{gw_1}{\sigma}+\left[\frac{w_1}{L}-\frac{gL}{\sigma}\tanh\left(\frac{w_1}{L}\right)\right]\right\}\left\{\left[\tanh\left(\frac{w_1}{L}\right)+\frac{\sigma L}{g}\right]\frac{\sigma' L}{\sigma L'}\tanh\left(\frac{w_2}{L'}\right)+\left[1+\frac{\sigma}{Lg}\tanh\left(\frac{w_1}{L}\right)\right]\right\}^{-1}\right]\right]. \tag{16}$$

For very low frequencies,

$$\tanh\left(\frac{w_1}{L}\right) \approx \frac{w_1}{L}, \qquad \tanh\left(\frac{w_2}{L'}\right) \approx \frac{w_2}{L'}, \tag{17}$$

$$\frac{\cosh(w_1/L)-1}{\cosh(w_1/L)} \approx \frac{1}{2}\frac{w_1^2}{L^2}. \tag{18}$$

Reducing equation (16),

$$S_{\Delta T_{\mathrm{av}}}(f) \propto \frac{1}{1+(\omega^2/\omega_0^2)} \propto \frac{1}{f^2+f_0^2}\,, \tag{19}$$

which is the low-frequency Lorentzian spectrum observed in the ACRIM data. The crossover frequency as a function of the constants chosen for the model is

$$f_0 = \frac{g}{2\pi[(w_1 c\rho + w_2 c'\rho'(1+gw_1/\sigma)]} \approx \frac{\sigma}{2\pi w_1 w_2 c'\rho'} \approx \frac{1}{8\ \text{months}}\,, \tag{20}$$

which is within an order of magnitude of the observed crossover frequencies of the 1987 and 1985 ACRIM data, $f = 1/(5\ \text{months})$ and $f = 1/(1\ \text{month})$, respectively.

At low frequencies,

$$\tanh\left(\frac{w_1}{L}\right) \approx \frac{w_1}{L}, \qquad \tanh\left(\frac{w_2}{L'}\right) \approx 1\,, \tag{21}$$

$$\frac{\cosh(w_1/L)-1}{\cosh(w_1/L)} \approx \frac{1}{2}\frac{w_1^2}{L^2}\,; \tag{22}$$

then

$$S_{\Delta T_{\mathrm{av}}}(f) \propto \frac{1}{2}\left(\frac{2gw_1}{\sigma}\right)^{1/2}\left(\frac{c\rho\sigma}{c'\rho'\sigma'}\right)^{1/2}\left(\frac{g}{w_1\rho c f}\right)^{1/2} \propto f^{-1/2}\,, \tag{23}$$

as observed.

The high- and low-frequency spectra meet at

$$f_1 = \frac{g}{w_1\rho c}\left(\frac{\sigma}{2gw_1}\right)^{1/3}\left(\frac{c'\rho'\sigma'}{c\rho\sigma}\right)^{1/3} 4^{1/3}\left(\frac{c\rho w_1}{c'\rho' w_2}\right)^{4/3} \approx \frac{1}{6\ \text{hr}}\,, \tag{24}$$

which also agrees to within an order of magnitude with the crossover frequency of the ACRIM data, $f = 1/(1\ \text{day})$.

We have applied the same model to the natural variability of climate (Pelletier 1995). The above model gives the fluctuations in the average temperature in a thin, homogeneous layer (Earth's atmosphere), coupled with another homogeneous layer with different thermal and diffusion constants (the ocean). As part of that work, we analyzed the Vostok ice core. We found the same spectral form reported here in the ACRIM data. The thermal and radiative timescales in that data set were 2000 and 40,000 yr, respectively. Estimates of those timescales based upon thermal constants and diffusion constants inferred from tracer studies in the atmosphere and ocean matched well the timescales of the crossover frequencies of the Vostok data.

4. CONCLUSIONS

We have presented evidence that the power spectrum of variations in solar irradiance exhibits three scaling regions. We presented a model, originally from van Vliet et al. (1980), proposed to study temperature fluctuations in a metallic film (granulation layer) supported by a substrate (deep convection zone) that matches the observed frequency dependence of the power spectrum of irradiance fluctuations.

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