

## Persistent drainage migration in a numerical landscape evolution model

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[1] Numerical landscape evolution models driven by uniform vertical uplift often develop a static drainage network and a precise balance between uplift and erosion. Small-scale physical models of uplifting drainage basins, however, continually reorganize by ridge migration and do not reach an ideal steady state. Here I show that the presence or absence of persistent drainage migration in a bedrock numerical landform evolution model depends on the flow-routing algorithm used to determine upstream contributing area. The model version that uses steepest-descent routing achieves an ideal steady state, while the model version that uses bifurcation routing results in continually-evolving drainage basins, even under conditions of uniform vertical uplift, bedrock erodibility, precipitation, and landsliding threshold. This result suggests that persistent drainage migration can occur by erosional processes alone. This result has important implications for numerical-modeling methodology, our understanding of the natural variability of landform evolution, and the interpretation of thermochronological data. *INDEX TERMS:* 1824 Hydrology: Geomorphology (1625); 3210 Mathematical Geophysics: Modeling; 8110 Tectonophysics: Continental tectonics—general (0905). *Citation:* Pelletier, J. D. (2004), Persistent drainage migration in a numerical landscape evolution model, *Geophys. Res. Lett.*, 31, L20501, doi:10.1029/2004GL020802.

### 1. Introduction

[2] Three-dimensional numerical modeling has become an important technique for understanding the coupled tectono-geomorphic evolution of mountain belts. One of the most successful ideas to originate from numerical modeling is the concept of steady-state mountain belts: rapidly-uplifting mountains may increase in relief until the topography and exhumation become time-independent [Stuwe *et al.*, 1994; Willett, 1999; Willett *et al.*, 2001; Batt, 2001]. At the scale of an entire mountain belt, a balance between uplift and erosion in rapidly-converging orogens is strongly supported by multiple lines of evidence [Willett and Brandon, 2002]. Whether or not ridges and valleys become “fixed” in the landscape, however, is more controversial. In models driven by purely vertical uplift, an ideal steady-state condition is often reached in which erosion and uplift are balanced at each point in the landscape [Willgoose *et al.*, 1991; Tucker and Slingerland, 1994; Howard, 1994; Kooi and Beaumont, 1996; Densmore *et al.*, 1998]. In models driven by horizontal tectonic

motion, drainages migrate, but this migration is primarily associated with tectonic motion [Willett, 1999; Willett *et al.*, 2001]. Small-scale physical models, in contrast, exhibit persistent drainage migration even under conditions of uniform vertical uplift and runoff [Hasbargen and Paola, 2000, 2003; Hasbargen, 2002]. Field and thermochronologic evidence also suggest that drainages continually migrate in both rapidly uplifting and inactive mountain belts [e.g., Burchfiel *et al.*, 2000; Naeser *et al.*, 2001]. Do numerical models underestimate the extent of drainage migration?

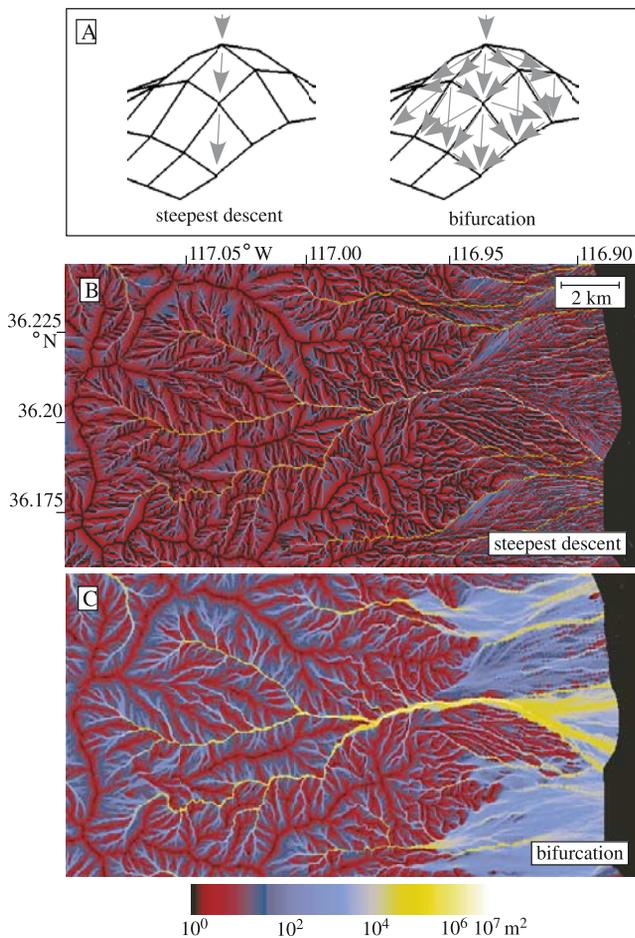
### 2. Bedrock Channel Incision and Flow Routing Algorithms

[3] The foundation of bedrock landform evolution modeling is the stream-power law, which in its basic form is written as

$$\frac{\partial h}{\partial t} = U - KA^m \left| \frac{\partial h}{\partial x} \right|^n, \quad (1)$$

where  $h$  is elevation,  $t$  is time,  $U$  is uplift rate,  $K$  is the coefficient of bedrock erodibility,  $A$  is contributing area,  $x$  is the along-channel distance, and  $m$  and  $n$  are exponents [Whipple and Tucker, 1999]. In addition to (1), bedrock landform evolution models also include hillslope evolution by diffusive processes and/or landsliding.

[4] Many variations of (1) have been considered in the literature, including thresholds for erosion and stochastic distributions of storms [Snyder *et al.*, 2003]. In comparison, little attention has been paid to effects of varying the method for calculating  $A$ .  $A$  has most often been estimated using the steepest-descent method, which routes all flow entering each grid point to the lowest neighboring grid point. This is a simple, widely-used flow-routing method, but it is not the most realistic method. Freeman [1991], for example, developed an alternative “bifurcation” method that routes flow to multiple downslope directions, weighted by bed slope. This algorithm is implemented by first ranking all grid points in the basin from highest to lowest in elevation. Starting with the highest grid point in the basin, which receives only local runoff, flow is distributed to all of the neighboring downslope grid points, weighted by slope. Routing is next performed for the second-highest grid point in the basin, then proceeding in rank order down to the lowest grid point. This method ensures that all incoming flow has been accounted for before the flow is distributed downstream. The bifurcation method is particularly useful for modeling distributed flow across low-relief surfaces, and it has played an important role in recent



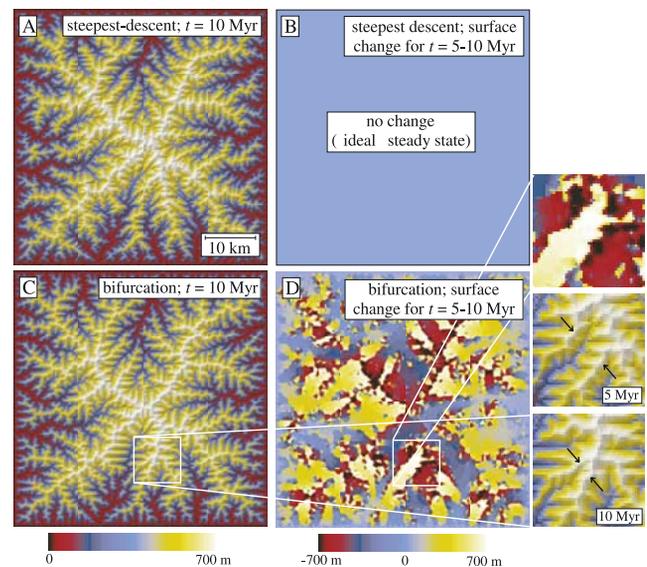
**Figure 1.** Comparison of steepest-descent and bifurcation flow-routing algorithms. (a) Steepest descent: a unit of precipitation that falls on a grid point (shown at top) is successively routed to the lowest of the eight nearest neighbors (including diagonals) until the outlet is reached. Bifurcation: all incoming flow to a grid point is distributed between the downslope pixels, weighted by bed slope. In this scheme, all the grid points in the basin are first ranked, and then flow routing proceeds from the highest to the lowest grid point. (b) Color map of contributing area calculated with steepest descent for Hanaupah Canyon, Death Valley, California, using USGS 30 m DEMs. Color scale is logarithmic and follows the legend at figure bottom. (c) Color map of contributing area computed with bifurcation routing, for the same area and color scale as (b). Bifurcation routing results in a more realistic flow distribution, particularly for hillslopes and areas of distributary flow.

numerical models of alluvial-fan evolution [Coulthard *et al.*, 2002; Clevis *et al.*, 2003]. Even in high-relief terrain, however, the bifurcation method results in substantially different flow distributions than those predicted by the steepest-descent method. Figures 1b and 1c, for example, illustrate the flow distributions for Hanaupah Canyon, Death Valley, using the steepest-descent and bifurcation methods, respectively. These algorithms result in entirely different flow distributions on the alluvial fan, as expected, but significant differences also exist on hillslopes and in

channels. Both flow-routing methods are approximations to the true distribution of flow, but the bifurcation method is more realistic because it is sensitive to both the drainage-network topology and hypsometry, while the steepest-descent method is sensitive to topology only. For example, the steepest-descent method does not distinguish between gentle, broad valleys and steep, narrow valleys with the same drainage network, but the bifurcation method results in more concentrated runoff in narrower valleys.

### 3. Model Description and Results

[5] In order to determine the influence of flow-routing methodology on the long-term behavior of numerical landform evolution models, I constructed a bedrock landform evolution model that solves (1) on a square grid subject to uniform vertical uplift and constant sea-level boundary conditions (e.g., Figure 2a). The model was implemented with a choice of steepest-descent or bifurcation routing. Landsliding was also considered: during each time step, slopes were compared to a predefined threshold value,  $S_c$ . Excess material was removed from upslope grid points until the threshold angle was achieved, checking slopes progressively from low elevations to high elevations in the grid. In this way, a single sweep through the grid stabilized all slopes.



**Figure 2.** Model results for steepest descent versus bifurcation flow routing. Parameter values:  $U = 1$  m/kyr,  $K = 7.5 \times 10^{-3}$  kyr $^{-1}$ ,  $m = 0.5$ ,  $n = 1$ ,  $S_c = 0.58$  ( $30^\circ$ ),  $L = 50$  km, and  $\Delta x = 250$  m. (a) Shaded-relief topography and (b) difference map ( $t = 5$ – $10$  Myr) for steepest-descent routing and (c)–(d) bifurcation routing. The drainage network at  $t = 10$  Myr is broadly similar in the two experiments ((a) and (c)). The steepest-descent model, however, achieves an ideal steady state by  $t = 5$  Myr (no topographic change is observed between  $t = 5$  and  $t = 10$  Myr), while ridges in the bifurcation model continue to migrate, as shown by the difference map in (d). A small portion of the difference map is highlighted, illustrating the contraction of a central drainage basin due to competition from two neighboring basins undergoing expansion.

[6] Model results are shown in Figure 2 for  $U = 1$  m/kyr,  $K = 7.5 \times 10^{-3}$  kyr $^{-1}$ ,  $m = 0.5$ ,  $n = 1$ ,  $S_c = 0.58$  ( $30^\circ$ ),  $L = 50$  km, and  $\Delta x = 250$  m ( $N \times N = 200 \times 200$ ), where  $L$  is the length of the basin and  $\Delta x$  is the pixel size. Slope-area relationships for bedrock rivers indicate that  $m/n \approx 0.5$  [Whipple and Tucker, 1999; Snyder *et al.*, 2000], and the linear case of  $n = 1$  was chosen for simplicity, thereby giving  $m = 0.5$  and  $n = 1$ . Figures 2a and 2c illustrate the model topography in shaded relief at  $t = 10$  Myr for steepest-descent and bifurcation routing, respectively. Figures 2b and 2d illustrate the change in topography, or difference map, between  $t = 5$  and  $t = 10$  Myr for both flow-routing methods. The model version with steepest-descent routing achieves an ideal steady state just prior to 5 Myr (at which point 7 relief units of erosion have taken place), and hence exhibits no change between 5 and 10 Myr. Bifurcation routing, in contrast, results in continually-migrating ridge lines, as shown by the difference map in Figure 2d. To help interpret this map, a portion of the map is highlighted, with the topography at  $t = 5$  Myr and  $t = 10$  Myr also shown for that region. Areas that are white and yellow in the difference map indicate drainages that have contracted in response to competition from neighboring drainages. Black and red areas, in contrast, have been more deeply dissected in response to expansion of the local drainage network and the resulting increase in stream power. The change map indicates that the change in local relief is of the same order as the total basin relief: a maximum of nearly 700 m. The bifurcation model run was carried out to 50 Myr, at which time ridge migration was still occurring. The persistence of drainage migration observed with bifurcation routing is a robust feature and occurs for a wide range of model parameter values.

[7] Bifurcation routing leads to enhanced drainage migration because it is continuously sensitive to the local topographic shape, while steepest-descent routing is sensitive only to the drainage-network topology. In models with steepest-descent routing, only the local slope can adjust to minimize differential erosion; the distribution of flow cannot adjust. Bifurcation routing, however, introduces an extra degree of freedom in the model because both the slope and flow distribution respond to the hypsometry or topographic shape. This extra degree of freedom introduces autogenic variability in the model dynamics and prevents equilibrium conditions from being precisely achieved.

#### 4. Discussion and Conclusions

[8] The model results shown in Figure 2 illustrate that drainage migration is a persistent feature of mountain-belt evolution even under conditions of uniform vertical uplift, bedrock erodibility, precipitation, and landsliding threshold. Models based on the steepest-descent method, therefore, can artificially “freeze” drainages into the landscape, either in an absolute reference frame (for models with purely vertical uplift) or in a moving tectonic reference frame (for models with horizontal tectonic motion). This result has important implications for the interpretation of thermochronologic data within tectono-geomorphic numerical models because the cooling history of a rock depends on the local exhumation rate and on the thermal boundary conditions (i.e., topography) experienced by the exhuming

rock. The results of this paper suggest that the thermal boundary conditions experienced by exhuming rocks may be more complex and time-dependent than those predicted by previous models that incorporated steepest-descent routing.

[9] The results of this paper also highlight the importance of drainage adjustment in the natural variability of landform evolution. Internal drainage adjustment is often overlooked as a mechanism for sediment pulses and stream terraces in favor of external forcing mechanisms such as climate change and episodic tectonism. For example, most proposed mechanisms for the Miocene sediment pulse from the Appalachians are climatic or tectonic in nature [e.g., Barron, 1989; Pazzaglia and Brandon, 1996]. Mechanisms based on drainage adjustment have not been proposed until recently, perhaps because of the difficulty of envisioning recent adjustments in a mountain belt that has been largely inactive since the Paleozoic. Thermochronologic evidence, however, supports the hypothesis that the Miocene sediment pulse was triggered by major drainage reorganization in which rivers west of the Blue Ridge reversed course [Naeser *et al.*, 2001]. Although the model of this paper does not reproduce dramatic shifts in drainage direction, adjustment is likely to be more vigorous under conditions of variable structure and lithology than for the uniform conditions assumed here. Piedmont terraces in the western U.S. are another example of a landform type that is usually interpreted as either climatic or tectonic in origin [Bull, 1991], although internal drainage adjustments are clearly important in some cases [Ritter, 1972]. Stan Schumm and his students have long emphasized the importance of autogenic mechanisms in controlling landscape evolution [Schumm and Parker, 1973; Parker, 1977]. Modern numerical models have generally not supported Schumm’s view. The results of this paper, however, suggest that numerical models have provided us with an artificially-static view of landscape evolution. As numerical models continue to improve, lateral migration of ridges should be considered an essential process in any numerical model, and as a valid hypothesis for episodic drainage-basin evolution.

[10] Although bifurcation routing is an improvement upon the steepest-descent algorithm, the model of this paper makes simplifying assumptions that likely underestimate the extent of lateral drainage migration in nature. Most importantly, channel flow is not resolved in cross section, and hence the model does not reproduce significant lateral valley migration in addition to ridge migration. In order to resolve valley migration, a multi-scale model that treats hillslopes at a coarse scale and channels at a much finer scale is most likely required. Some models have this capability [e.g., Tucker *et al.*, 2001], but there are formidable computational challenges in simulating large basins over geologic time in these models. As such, new computational approaches (e.g., front-tracking methods [Sethian, 1996]) may be necessary.

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